GrEnFIn

Greening Energy
Market and Finance

University
of Economics
in Katowice

The transition risk and the regulations: Green bonds and beyond

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Schedule

1. Bond – main features
2. Green Bond characteristics
3. Green Bond market
4. Bond’s price and yield relation
5. Coupon calculation of a new issue
6. Measures of bond’s risk

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Part 1

BONDS
MAIN FEATURES

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Bond - definition

Bond –a security that is usually issued by corporations or governments for the purpose of borrowing funds.

Those who purchase bonds expect the issuing party to repay the principal on the future date (**par value**, face value) and the interest in the form of periodic payments (**coupon payments**)

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Bond indenture

- ✓ date of issue
- ✓ maturity date (redemption date)
- ✓ nominal (par value, face value)
- ✓ coupon rate (and its frequency)
- ✓ yield rate
- ✓ price

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Conventional bond

Conventional bond (plain vanilla, bullet)

- ✓ periodic payments (paid at regular intervals) are based on a fixed income coupon
- ✓ at maturity the buyer receives a nominal

All other papers are a variation of conventional securities

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Types of bonds - Coupon

Coupon

- ✓ fixed (fixed income assets)
- ✓ zero (zero-coupon papers)
- ✓ benchmark (floating rate notes, FRN)
- ✓ index (index-linked bonds)

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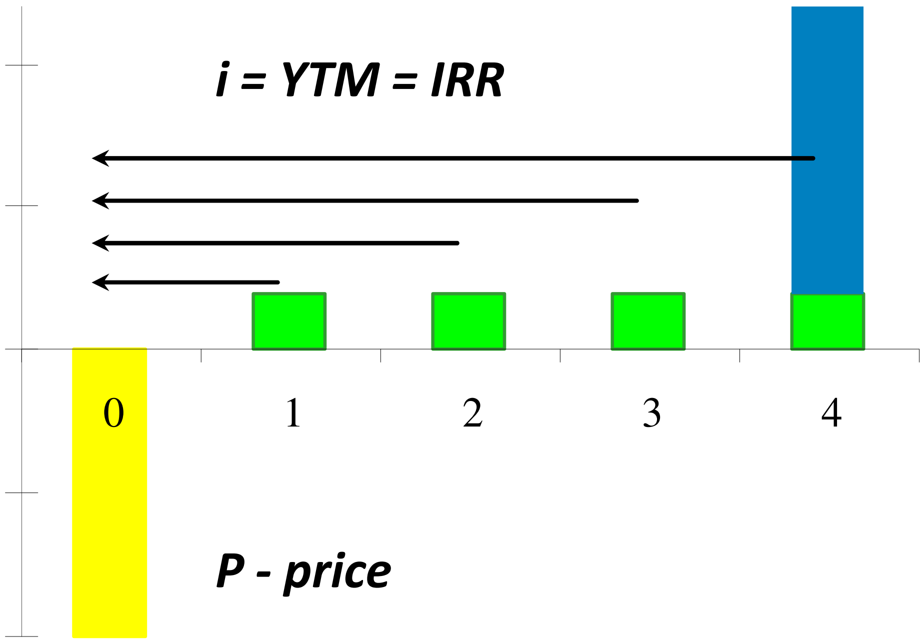
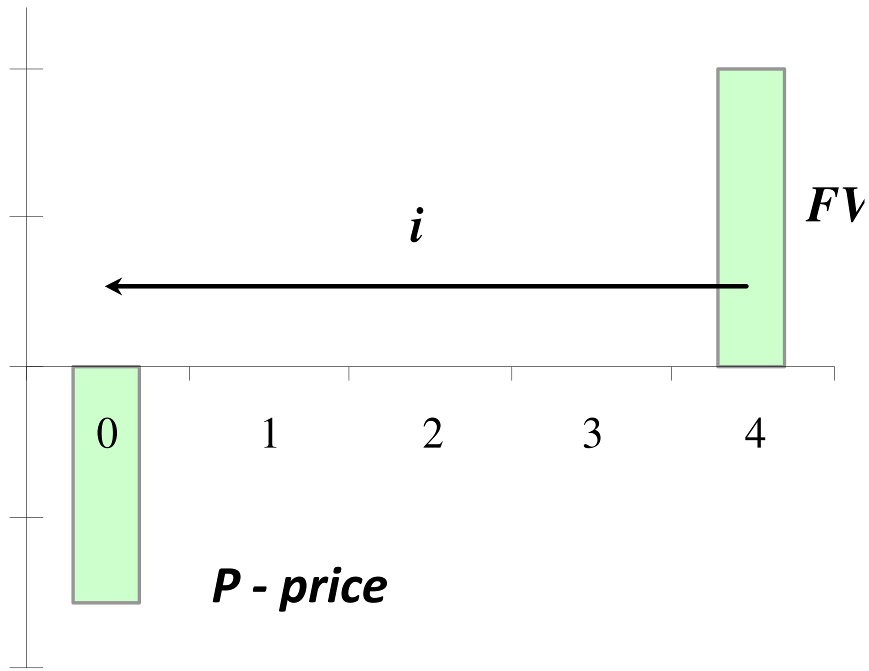
Coupon vs zerocoupon bond





Types of bonds - Issuer

Issuer

- ✓ government (treasury bonds, sovereign bonds)
- ✓ corporates – corporate bonds
- ✓ supranational entities (World Bank, EBI)
- ✓ regions and municipalities
- ✓ projects and SPVs

GREEN BOND CHARACTERSISTICS

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Sustainable finance

Sustainable finance is defined as the incorporation of environmental, social, and governance (ESG) principles into business decisions, economic development, and investment strategies.

Key Pillars	Key Themes
Environment	Climate change
	Natural resources
	Pollution and waste
	Opportunities and policy
Social	Human capital
	Product responsibility
	Relations
Governance	Corporate governance
	Corporate behavior

Application of ESG factors across Asset Classes

Asset Class	Type
Equity	Negative (exclusionary) and positive (best-in-class) approach
Debt Fixed Income	Traditional corporate bonds
	Traditional sovereign bonds
	ESG money market funds
	Green bonds
	Social bonds
	Sustainability bonds
	Green mortgage-backed securities (MBS)
Debt Bank Loans	Green loans
	Sustainability-linked loans
Alternative Investment	Green real estate investment trusts (REIT)
	Private equity (PE) and venture capital (VC)

Application of ESG factors across Asset Classes

Debt Fixed Income	Examples
Traditional corporate bonds	Bonds with proceeds used for funding new and existing projects with environmental benefits (not labeled).
Traditional sovereign bonds	Bonds issued by governments with proceeds used for funding projects with environmental benefits (not labeled).
Green bonds	Specific bonds that are labeled green, with proceeds used for funding new and existing projects with environmental benefits.
Social bonds	Bonds that raise funds for new and existing projects that create positive social outcomes.
Sustainability bonds	Bonds with proceeds that are used to finance or refinance a combination of green and social projects.
Green mortgage-backed securities (MBS)	Green MBS securitize numerous mortgages that go toward financing green properties (Fannie Mae)

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Green bonds

Green bonds are fixed income securities which finance investments with environmental or climate-related benefits.

Specifically, these bonds have the following features:

- ✓ proceeds are allocated exclusively to green projects,
- ✓ proceeds are tracked and managed in a reliable manner, and
- ✓ transparency is ensured by reporting after the issuance of the bonds.

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Green projects

Green bonds have been used to finance:

- ✓ climate projects,
- ✓ pollution prevention,
- ✓ sustainable agriculture,
- ✓ sustainable water management and
- ✓ other environmental initiatives.

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Sustainable Development Goals

1 NO POVERTY	Waste water treatment projects
12 RESPONSIBLE CONSUMPTION AND PRODUCTION	Waste management projects
7 AFFORDABLE AND CLEAN ENERGY	Wind farms and solar PV projects, green buildings
13 CLIMATE ACTION	Flood protection projects
9 INDUSTRY, INNOVATION AND INFRASTRUCTURE	Maintenance and upgrade of public transport and infrastructure projects
14 LIFE BELOW WATER	Renaturation projects
11 SUSTAINABLE CITIES AND COMMUNITIES	Low carbon public transport projects
15 LIFE ON LAND	Sustainable agriculture and forestry projects

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Types of Green Bonds (ICMA)

Green bond issuing entities can make use of a variety of structures related to the "use of proceeds"

- ✓ Green Use of Proceeds Bond:
- ✓ Green Revenue Bond:
- ✓ Green Project Bond:
- ✓ Green Securitized Bond:

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Types of Green Bonds (ICMA)

Type	Definition
Green Use of Proceeds Bonds	Similar to traditional bonds by offering full recourse to the issue and sharing the same credit rating as the issuer.
Green Use of Proceeds Revenue Bonds	Non-recourse to the issuer and repays investors based on a revenue stream such as tolls, fees, and taxes.
Green Project Bonds	Recourse or non-recourse to the issuer.
Green Securitized Bonds	Bond collateralized by one or more specific Green Project(s).The first source of repayment is generally the cash flows of the assets.

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Part 3

GREEN BOND MARKET

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Green bond market

https://www.elibrary.imf.org/doc/IMF082/26206-9781498324021/26206-9781498324021/Other_formats/Source_PDF/26206-9781513515946.pdf

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First green bond

https://www.youtube.com/watch?v=i3gIJrABLSc&index=1&list=PLuz9mCSZhLGklkMaOJTBKkiKwzoHFt6Sx

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Who decides what is green?

There is no standard for what kinds of activities can be funded by green bonds. Absent common standards or criteria, the vast majority of green bonds are self-labeled by the issuer.

- ✓ World Bank & CICERO
- ✓ ICMA --> Green Bond Principles
- ✓ Climat Bond Initiative --> Climat Bond Standards

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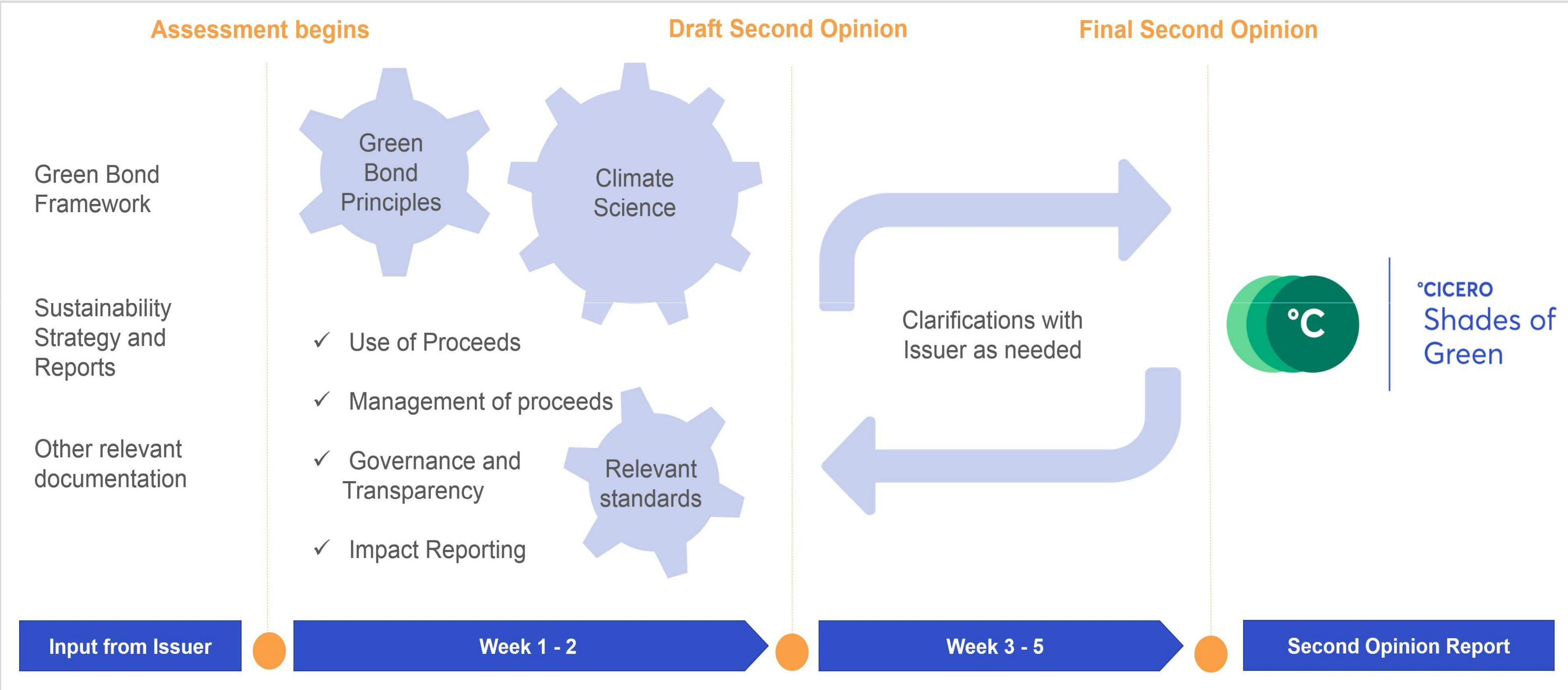
GrEnFIn World Bank & CICERO

The World Bank decides what projects can be eligible for green bond proceeds based on its own selection criteria.

These criteria were reviewed by the Center for International Climate and Environmental Research University of Oslo (CICERO). CICERO also certified the International Finance Corporation's criteria for green bonds.

The process of getting a CICERO Second Opinion on a green bond framework



Source: https://cicero.oslo.no/en/posts/single/CICERO-second-opinions

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CICERO Shades

SHADES OF GREEN AND BROWN	EXAMPLES
Dark green is allocated to projects and solutions that correspond to the long-term vision of a low carbon and climate resilient future.	Wind energy projects with a governance structure that integrates environmental concerns.
Medium green is allocated to projects and solutions that represent steps towards the long-term vision, but are not quite there yet.	Green buildings with a high level of certification and energy efficiency
Light green is allocated to projects and solutions that are environmentally friendly but do not by themselves represent or contribute to the long-term vision.	Hybrid personal vehicles
Light brown for efficiency improvements in projects that are associated with fossil fuel use but do not necessarily promote locking-in of emissions. Changes in the way assets are used may position them in the light green category.	Efficient fossil fuel cargo vessels
Medium brown projects can be lower emissions, but still represent risk of locking in fossil fuel infrastructure and are exposed to risk of stranded assets.	New infrastructure for natural gas
Dark brown for the heaviest emitting projects, with the most potential for lock-in of emissions and risk of stranded assets.	New infrastructure for coal

Source: https://www.cicero.green/latestnews/2020/5/13/launching-cicero-shades-of-green-assessment-for-companies-and-equities

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Climate Bonds Standards (CBS)

The Climate Bonds Initiative (CBI) is developing standards for a bond to be eligible for an industry-recognized label of “Certified Climate Bond.” It has developed a climate bonds taxonomy to establish common definitions for eight broad categories —energy, energy efficiency, transport, water, waste management, land-use and adaptation infrastructure — which are then further defined with criteria, explanations and restrictions.

https://www.climatebonds.net/files/files/CBI-Taxonomy-Sep18.pdf

https://www.climatebonds.net/standard/taxonomy

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Green Bond Principles (GBP)

A voluntary process guidelines ICMA that recommend transparency and promote integrity in the development of the Green Bond market by clarifying the approach for issuance of a Green Bond.

The GBP have **four core components**:
(1) the use of proceeds
(2) the process for project evaluation and selection
(3) the management of proceeds
(4) reporting

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The issuers of green bonds

- ✓ corporations that raise funds for Green Projects (including SPCs that only handle Green Projects),
- ✓ financial institutions that raise investment funds and loans for Green Projects, and
- ✓ local governments that raise funds for Green Projects.

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The investors of green bonds

- ✓ institutional investors, such as pension funds and insurance companies that commit to ESG (environmental, social, and governance) investments;
- ✓ investment managers entrusted with the management of ESG investments, and
- ✓ individual investors who focus on the use of the proceeds.

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Green bonds benefits

Investors	Issuers
Reputational benefits (e.g. marketing can highlight support for green investment);	
Articulation and enhanced credibility of sustainability strategy ("money where your mouth is");	
Lack of additional risk, green bonds can be incorporated into pension funds' existing asset allocations;	Access to "economies of scale" as majority of issuance costs are in setting up the processes;
Improving diversification of bond issuer base;	Improving diversification of bond investor base;

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Climate-related financial risks

Physical risks - arise from damage to property, land, and infrastructure from catastrophic weather-related events and broader climate trends;

Transition risks - arise from changes in the price of stranded assets and broader economic disruption because of evolving climate policy, technology, and market sentiment during the adjustment to a lower-carbon economy.

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Recommendations

Central Banks and Supervisors
Network for Greening the Financial System

1. Integrating climate-related risks into financial stability monitoring and micro-supervision
2. Integrating sustainability factors into own-portfolio management
3. Bridging the data gaps
4. Building awareness and intellectual capacity and encouraging technical assistance and knowledge sharing
5. Achieving robust and internationally consistent climate and environment-related disclosure
6. Supporting the development of a taxonomy of economic activities

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BIS - green bond fund

09.2019 - a green bond fund for central banks was launched to:

- ✓ incorporate environmental sustainability objectives into management of the central banks' foreign exchange reserves.
- ✓ finance environment-friendly projects.
- ✓ affect the development of green investment standards.
- ✓ deepen the green bond market and foster best practice.

The open-ended fixed income fund, denominated in US dollars, invests in green assets with a minimum grade of A–. To be eligible, bonds must conform to either the ICMA's **Green Bond Principles** or the CBI's **Climate Bond Standards.**

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Criteria

The researchers examined green bond suitability as currency reserves by looking at the three criteria of:

- ✓ liquidity,
- ✓ safety and
- ✓ return.

Liquidity could well restrict a central bank's allocation because the $750 billion in green bonds pales next to the $120 trillion in conventional securities.

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Part 4

BOND'S PRICE and YIELD RELATION

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Coupon rate vs yield rate

Coupon rate (bond rate (US), contract rate) – the rate used as a basis for computing coupon payments (it is stated on the bond)

Yield rate (investor's rate, yield to maturity, YTM) – the actual interest rate that is expected by the purchaser of the bond (is is usually not equal to the coupon rate)

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Coupon rate vs yield rate

	Coupon rate	Yield rate
Who establishes	Issuer	Investor / market
Meaning	How much the issuer promises to pay	How much the investor wants to achieve
Where used	Coupon calculation	Price calculation
Changable	no (for fixed income)	yes (market conditions)

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Purchase price of bond

<u>Purchase price</u> – price of the bond **P** computed at the yield rate.

It is the present value of the periodic payments which are paid from the date of purchase to the maturity.

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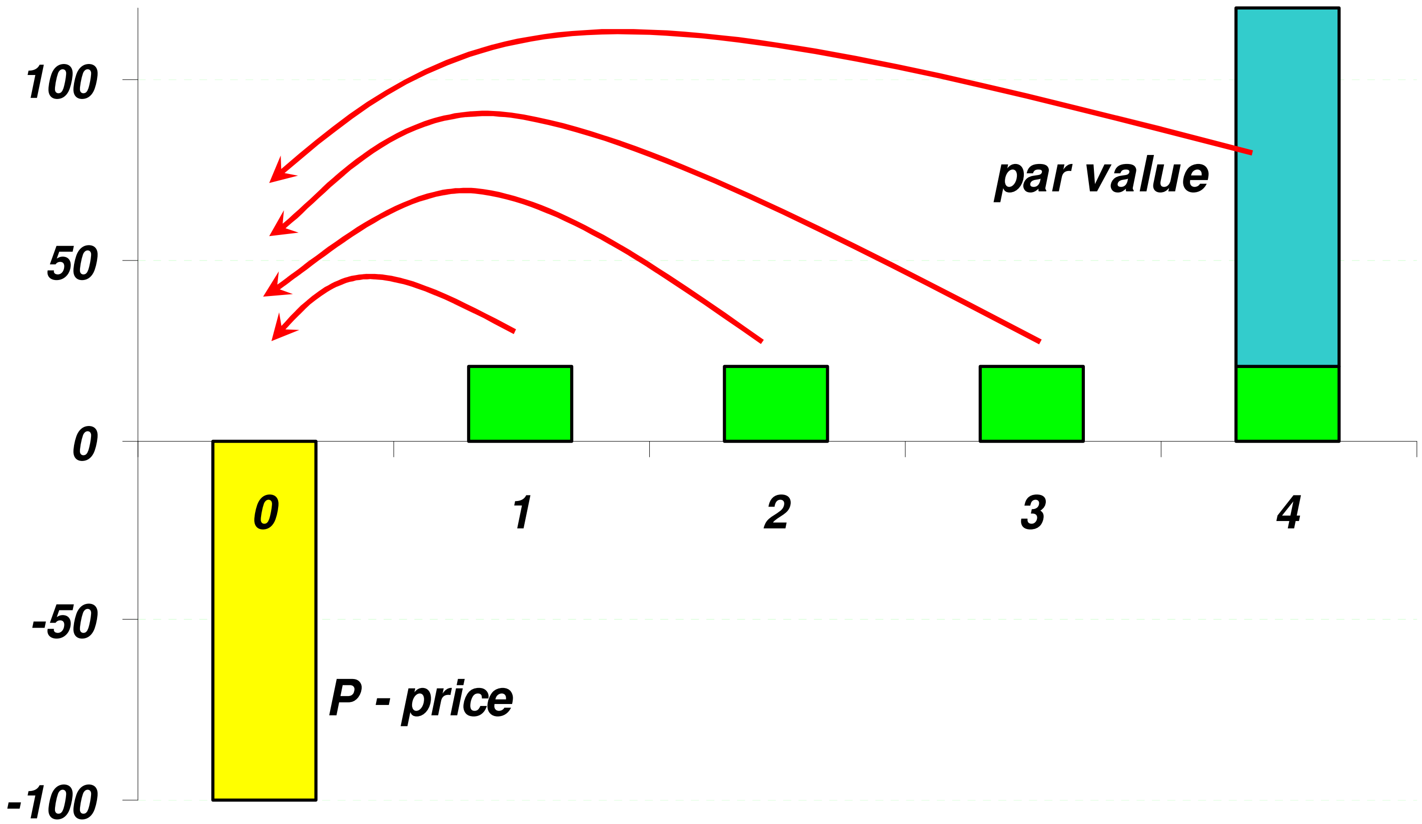
Purchase price of bond



Purchase price of bond

$$P = FV + FV \cdot (i_{cp} - i)\frac{1-(1+i)^{-N}}{i}$$

where : P – purchase price of bond

FV – face value, par value

i_{cp} – coupon rate

i – yield rate

N – number of payments (=*n for annually payments*)

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$$P = FV + FV \cdot (i_{cp} - i)\frac{1-(1+i)^{-N}}{i}$$

(1). If the investor wants bigger rate then the issuer offers:

$$i > i_{cp} \quad \Rightarrow \quad (i_{cp} - i) < 0 \quad and \quad P < FV$$

(2). If: $$i < i_{cp} \quad \Rightarrow \quad (i_{cp} - i) > 0 \quad and \quad P > FV$$

(3). If: $$i = i_{cp} \quad \Rightarrow \quad (i_k - i) = 0 \quad and \quad P = FV$$

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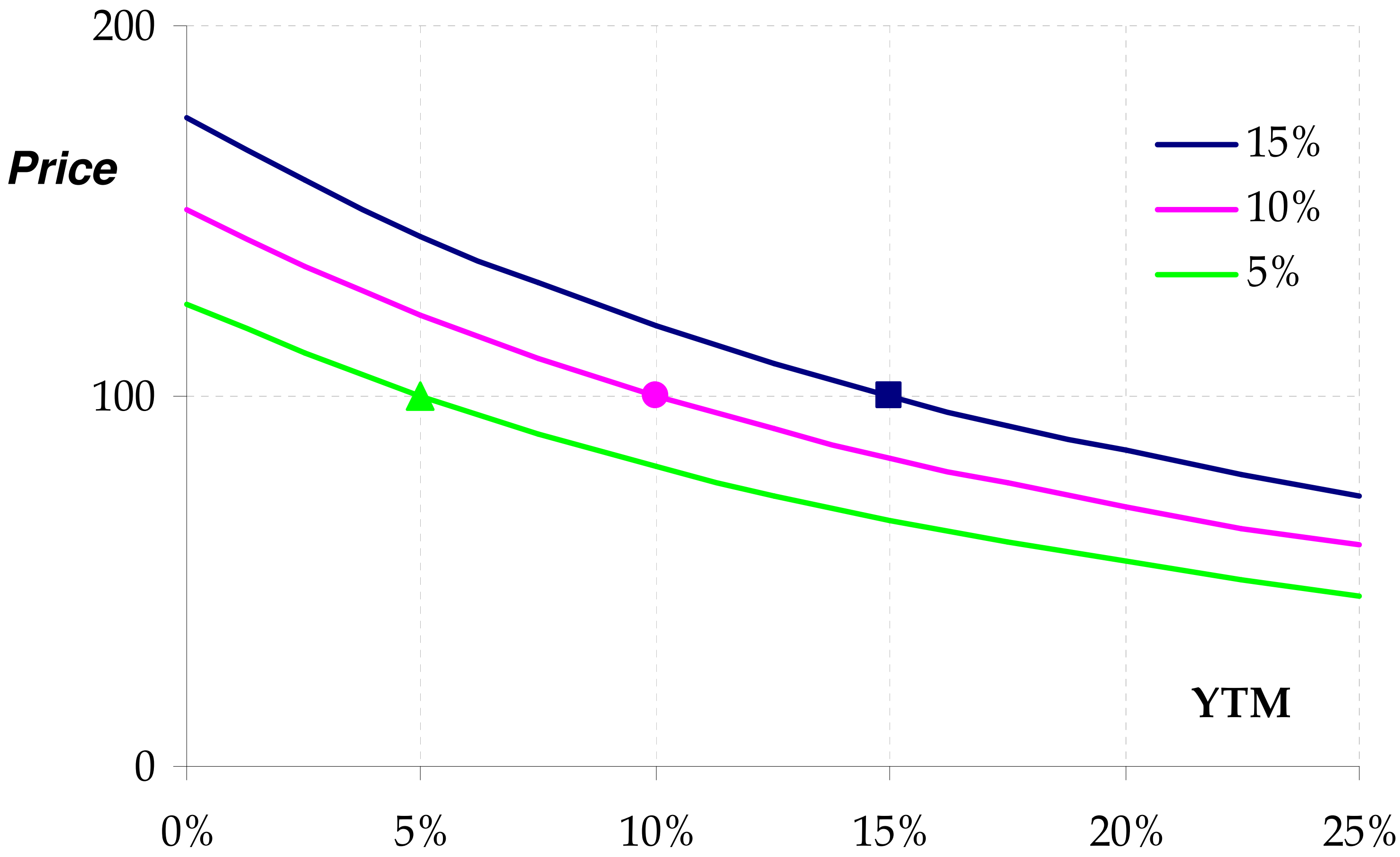
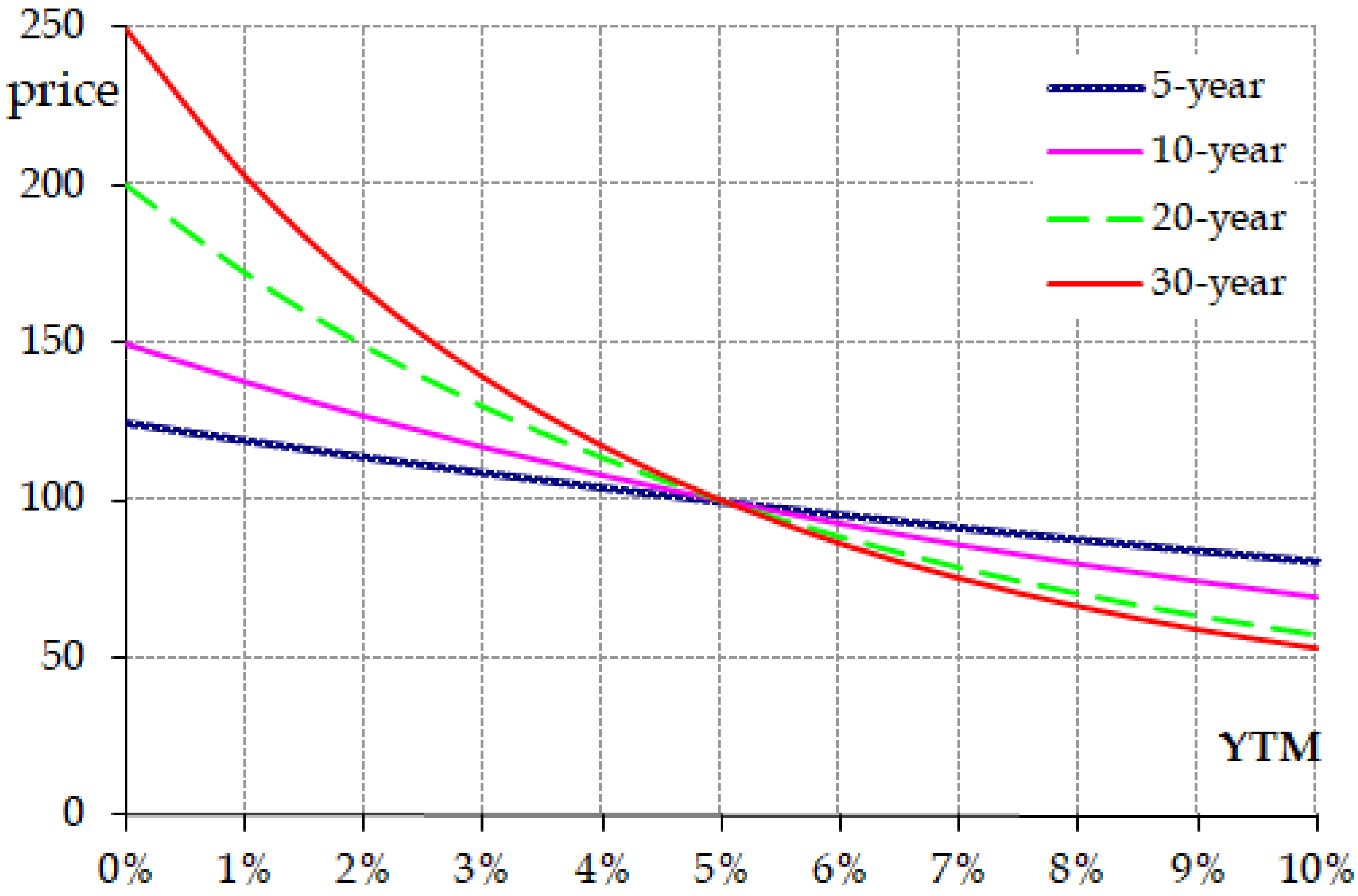
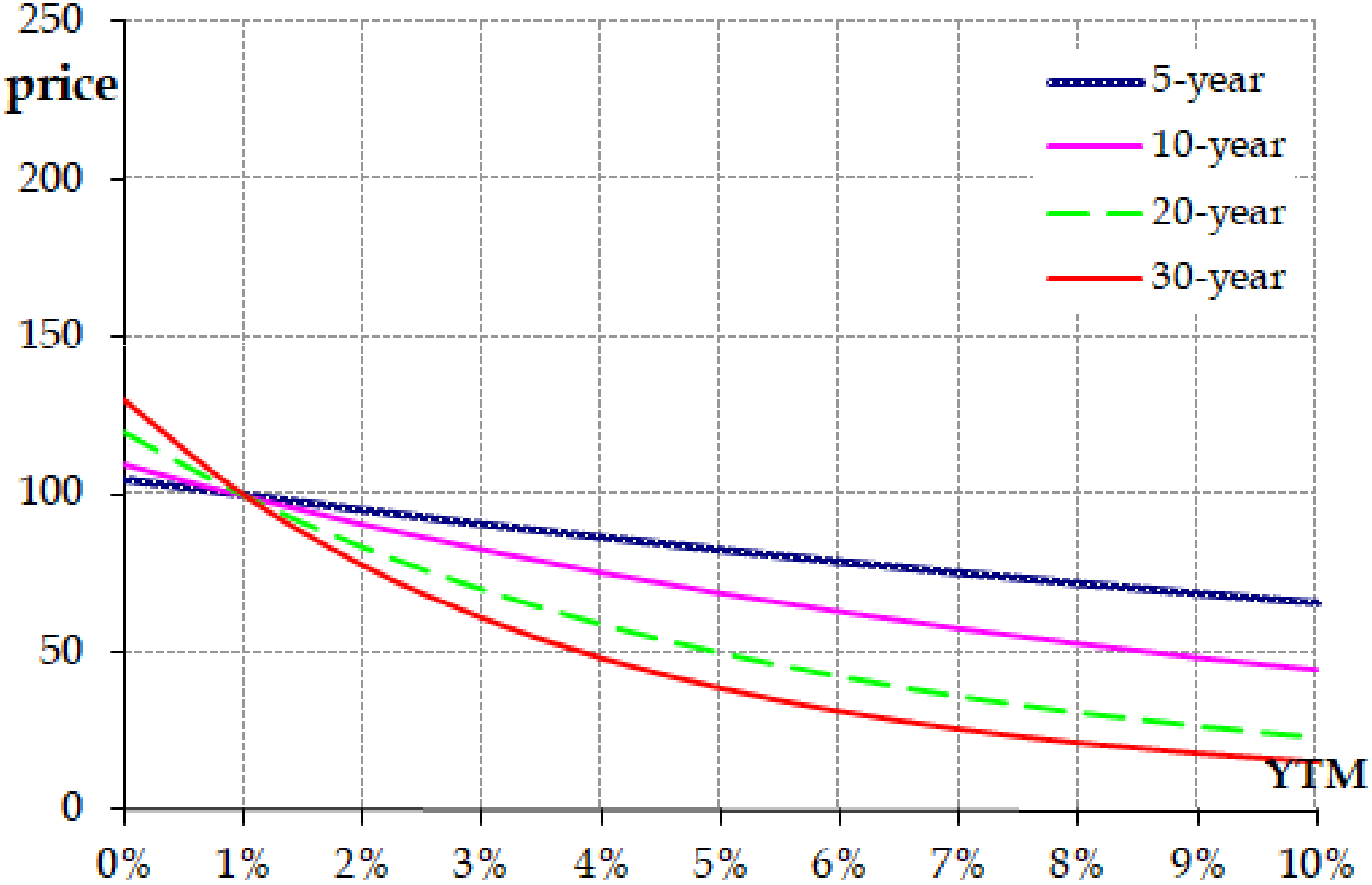
Price – yield relationship (5y-bond)



Price – yield relationship (5% coupon-bond)



Price – yield relationship (1% coupon-bond)



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Example 4.1

Find the purchase price of the 5-years green bond (par = 100) bought 3 years before maturity, if the coupon rate is 8% (paid annually), and the investor would like to achieve the yield at 9%.

$$P = FV + FV \cdot (i_{cp} - i)\frac{1-(1+i)^{-N}}{i} =$$

$$= 100 + 100(0,08 - 0,09)\frac{1-(1+0,09)^{-3}}{0,09} =$$

$$= 97,4687$$

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Excel calculations

Excel has a function that allows you to price straight bonds, and it is called PRICE.

=PRICE(“Today”,“Maturity”,Coupon Rate,YTM,100,2,3)

- Enter “Today” and “Maturity” in quotes, using mm/dd/yyyy format.
- Enter the Coupon Rate and the YTM as a decimal.
- The "100" tells *Excel* to us $100 as the par value.
- The "2" tells *Excel* to use semi-annual coupons.
- The "3" tells *Excel* to use an actual day count with 365 days per year. (1-ACT/ACT; 2-ACT/360; 3-ACT/365; 4-30/360)

Note: *Excel* returns a price per $100 face.

Semiannual coupon payments

The interest conversion period for the yield rate is assumed to coincide with the interest payment period for the bond

$$i \leftarrow \frac{i}{m}$$

$$i_{cp} \leftarrow \frac{i_{cp}}{m}$$

$$N = n \leftarrow n \cdot m$$

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Example 4.2

Find the purchase price of the 5-years green bond bought 3 years before maturity, if the coupon rate is 8% (paid semiannually), and the investor would like to achieve the yield at 9%.

$$i = \frac{i}{m} = \frac{0{,}09}{2} = 0{,}045 \qquad i_{cp} = \frac{i_{cp}}{m} = \frac{0{,}08}{2} = 0{,}04$$

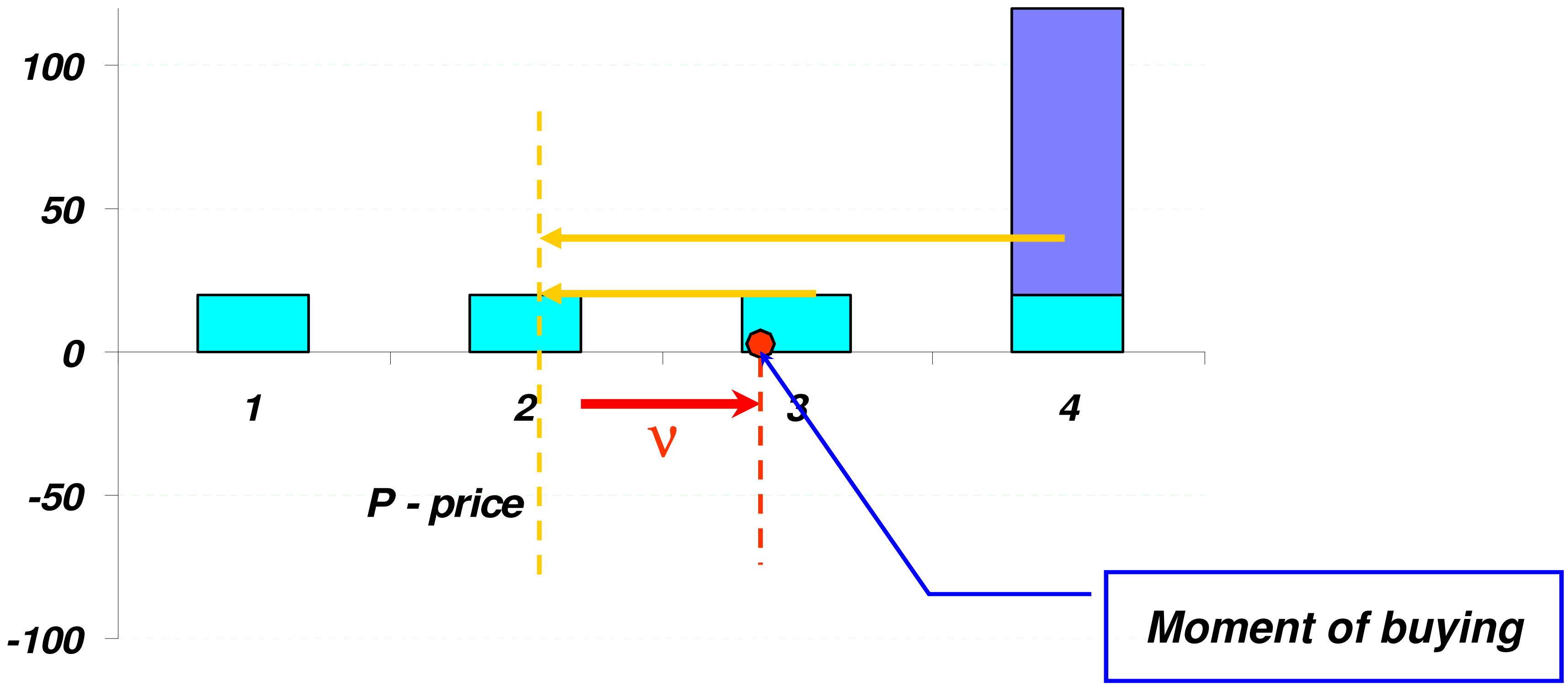
$$N = n \cdot m = 3 \cdot 2 = 6$$

$$P = FV + FV \cdot (i_{cp} - i)\frac{1-(1+i)^{-N}}{i} =$$

$$= 100 + 100(0{,}04 - 0{,}045)\frac{1-(1+0{,}045)^{-6}}{0{,}045} = \ 97{,}421$$

Purchase price of bond

To discount by number of full years and multiply by part of the coupon period.



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Purchase price of bond

$$P = \left[FV + FV \cdot (i_{cp} - i) \frac{1-(1+i)^{-N}}{i} \right] (1+i)^{v}$$

To discount by number of full years

To multiply by part of the coupon period.

where : P- price

i_c coupon rate

FV – face value, nominal

i - yield

N – number of coupon payments

v – time since the last coupon payment

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Base

$$v = \frac{\tau}{base}$$

where :

τ – number of days since the last coupon payment

base – number of days in a year

v – time (in years) since the last coupon payment

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Accrued interest

- **clean price flat price -** price without the accrued interest
- **dirty price, full price**, or **invoice price** - price with the accrued interest

- Accrued interest – AI

$$AI = C \cdot v$$

$$C = FV \cdot i_{cp}$$

Example 4.3

A bond with maturity at 17.06.2021 was bought on 8.07.2019. The coupon rate is 2%, nominal 100 EUR. Please calculate the price if the yield to maturity is 2.122% (base is ACT/ACT)

$$\nu = 21/366$$

$$P = \left[FV + FV \cdot (i_{cp} - i) \frac{1-(1+i)^{-N}}{i} \right] (1+i)^{\nu} =$$

$$= \left[100 + 100 \cdot (0{,}02 - 0{,}02122) \frac{1-(1{,}02122)^{-2}}{0{,}02122} \right] (1{,}02122)^{\frac{21}{366}}$$

$$P = 99{,}8838$$

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Example

A bond with maturity at 17.06.2021 was bought on 8.07.2019. The coupon rate is 2%, nominal 100 EUR. Please calculate the clean price and AI (base is ACT)

$$AI = 100 \cdot 0{,}02 \cdot \frac{21}{366} = 0{,}114754$$

$$P(clean) = 99{,}8838 - 0{,}1147 = 99{,}77$$

Zero-coupon bond

$$P_{zero} = \frac{FV}{(1+i)^n}$$

where : P- price of the bond

FV - nominal

i –yield to maturity

n – time to maturity **in years**

Example 4.4

Calculate a price of the zero-coupon bond if the YTM=5%, the bond was bought on 12.11.2019, its maturity is 25.07.2021, and the interest is calculated with ACT/365 rule

$$n = 621/365$$

$$P_{zero} = \frac{FV}{(1+i)^n} = \frac{100}{(1+0,05)^{\frac{621}{365}}}$$

$$P = 92,034$$

Example 4.5

Issuer	PepsiCo
Issuance date	October 2019
Nominal value	$1 billion
Nominal currency	USD
Rating (issuer, bond)	A+ (S&P), A (Moody's)
Framework	Green bond
Tenure	30 years
Coupon	2.875%
Use of proceeds	Eco-friendly plastics, water use efficiency, packaging, and cleaner transportation
Bookrunners	Morgan Stanley, Goldman Sachs, Mizuho Financial group

https://sec.report/Document/0001047469-19-005653/0001047469-19-005653.txt#ds41801_description_of_debt_securities

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Example 4.6

Issuer	U.S. State of Massachusetts
Issuance date	September 2014
Nominal value	$350 million
Nominal currency	USD
Rating (issuer, bond)	AA+ (Fitch), Aa1 (Moody's), AA+ (S&P)
Framework	Green bond
Tenure	3 to 17 years
Coupon	2.45%
Subscription level	3 times
Investor base	Residents and local retail investors
Use of proceeds	Water projects, offshore wind port facilities, energy-efficient buildings, and restoration and preservation projects
Bookrunners	Morgan Stanley

https://www.climatebonds.net/files/files/DC%20Water%20case%20study%20-%20final%281%29.pdf

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Example 4.7

https://www.wienerborse.at/en/market-data/bonds/quote/?ISIN=XS2126053953&ID_NOTATION=283722099

Find the YTM of the green bond with maturity on 03/05/2025, the coupon equal to 1.8%, paid annually, bought 8.06.2020 at 101.05 (offer) or sold at 99.1 (bid)

Answer: (offer) 1.58%

(bid) 1.99%

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Example 4.8

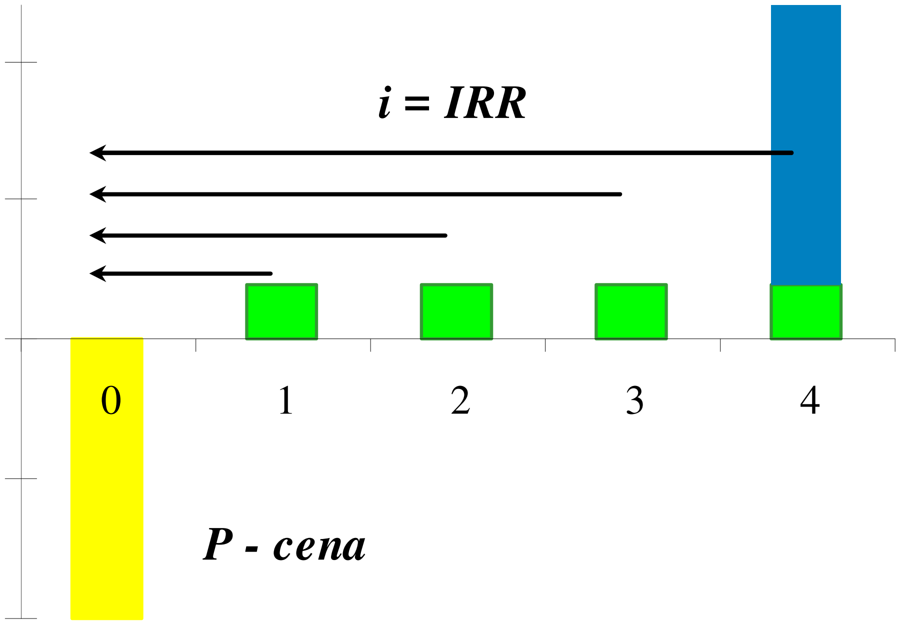
https://www.bondsupermart.com/bsm/bond-factsheet/XS1512652600

Find the YTM of the green bond with maturity on 25.01.2020, the coupon equal to 8.25%, paid semiannually, offered on issue date 25.01.2017 at 98.385

Answer: 8.87%

COUPON CALCULATION of NEW ISSUE

YTM for a coupon bond



$$YTM = IRR$$

where :

P – bond’s price

N - nominal

i – desired interest rate

n – time to maturity **in years**

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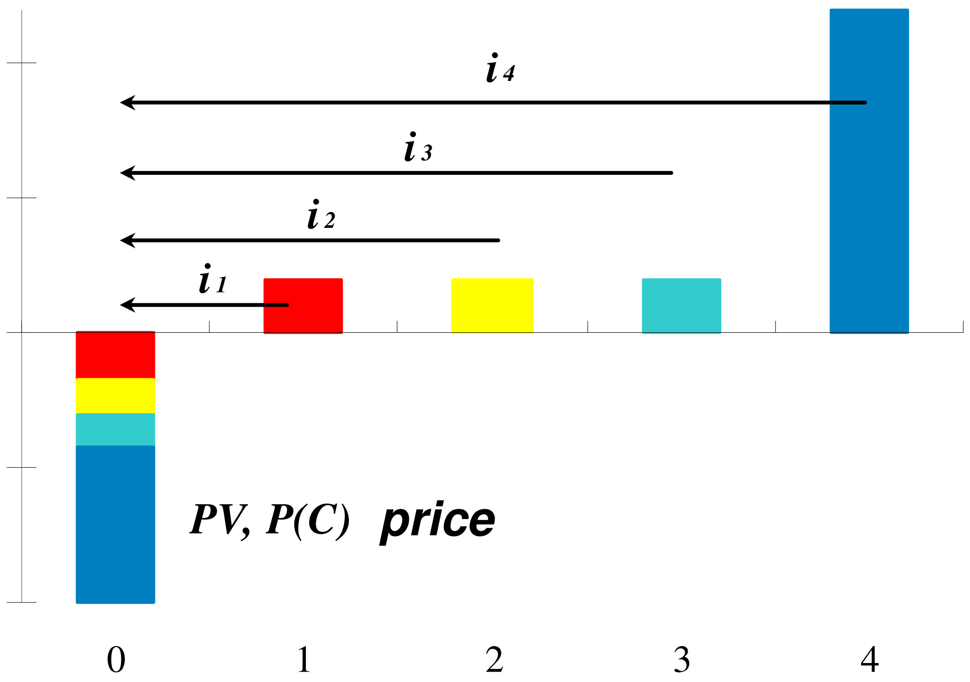
Coupon bond as a sum (portfolio) of zero-coupon bonds



where :

P – bond's price

N - nominal

i – desired interest rate

n – time to maturity **in years**

$$P(C)=\frac{C}{(1+i_1)^{t_1}}+\frac{C}{(1+i_2)^{t_2}}+\cdots+\frac{C+N}{(1+i_n)^{t_n}}$$

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Discount factors

- It is a present value of a unit of currency to be received at the end (of the term)
- For t-years is written d(t)

$$d(t_j) = \begin{cases} \dfrac{1}{(1+i_j)^{t_j}} \end{cases}$$

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Example 5.1

For a particular terms and spot rates find discount factors

	spot rate
0	
1	3,50%
2	3,70%
5	3,55%
10	3,45%

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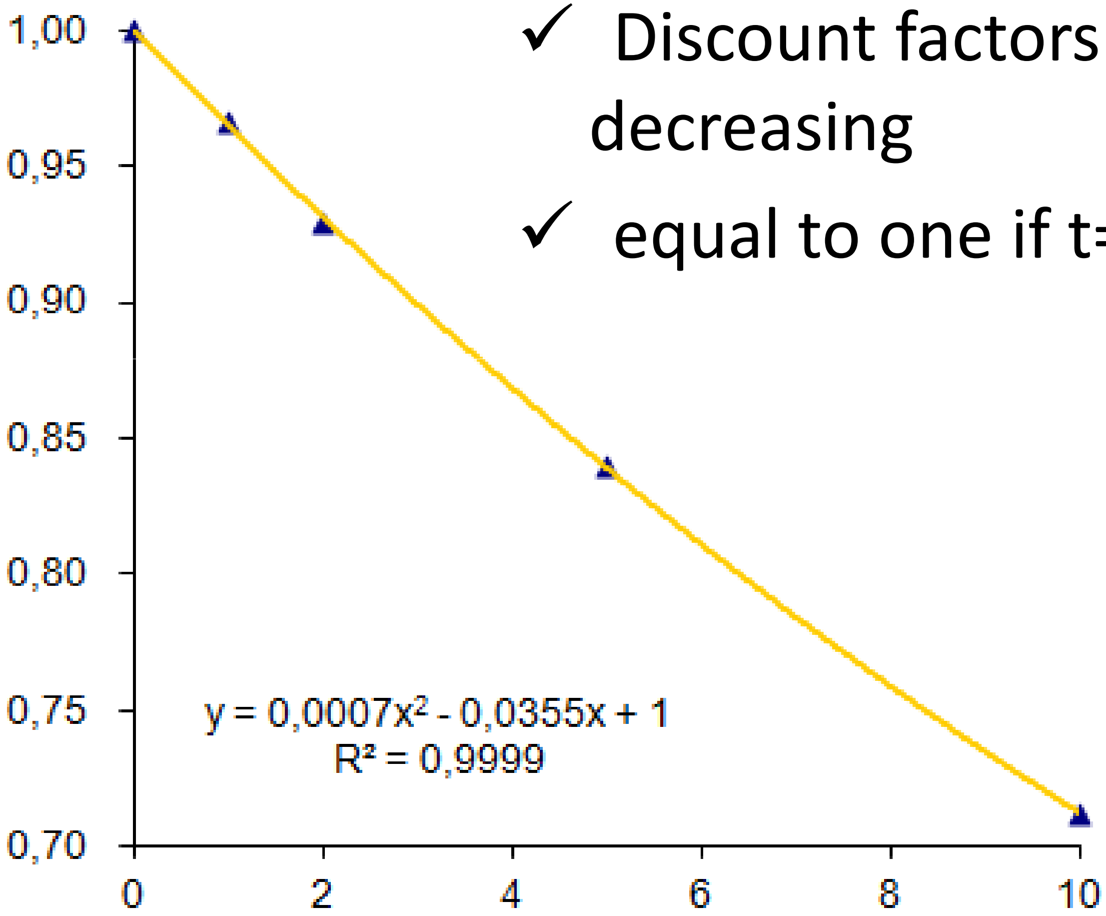
Discount function - features



- ✓ Discount factors fall with maturity - decreasing
- ✓ equal to one if t=0

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Coupon bond as a sum (portfolio) of zero-coupon bonds

$$P(C) = \frac{C}{(1+i_1)^{t_1}} + \frac{C}{(1+i_2)^{t_2}} + \cdots + \frac{C+N}{(1+i_n)^{t_n}}$$

$$P(C) = C \cdot d(t_1) + C \cdot d(t_2) + \cdots + (C+N) \cdot d(t_n)$$

where:

P(C) – price of a coupon bond

t_1, t_2...t_n – term of j-period (in years), where j=1,2,…n

$d(t_j)$ – discount factors for t_j, where j=1,2,…n

N – nominal

C – coupon (N x coupon rate),

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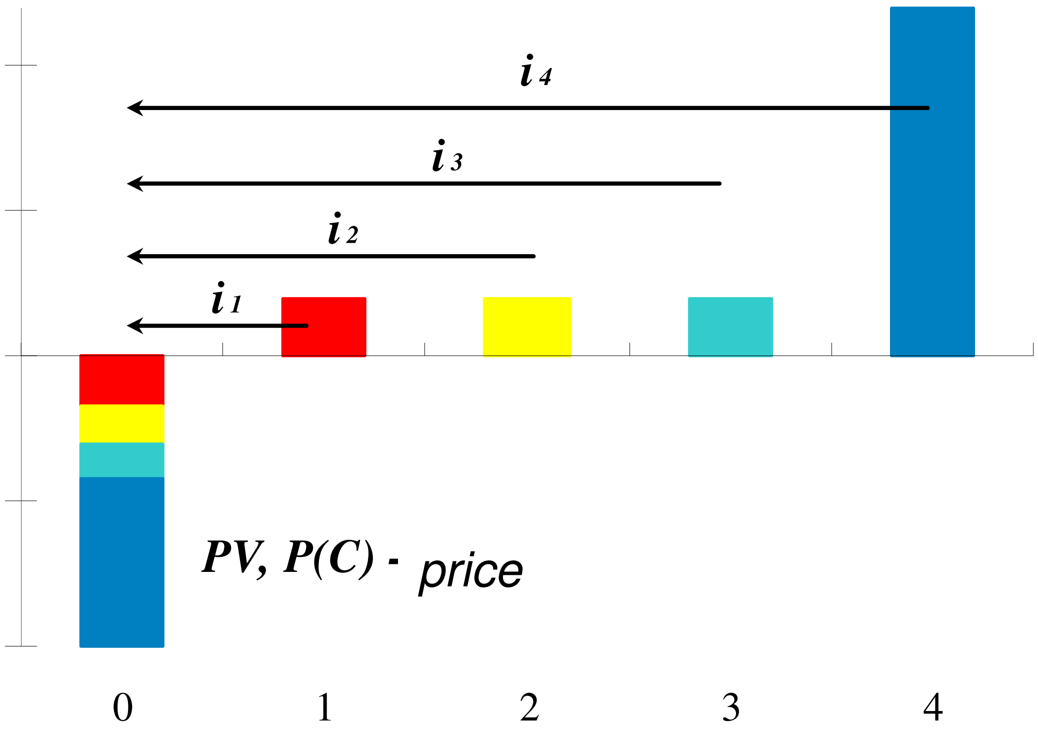
Coupon bond as a sum (portfolio) of zero-coupon bonds



where :

P – bond's price

N - nominal

– desired interest rate

n – time to maturity **in years**

$$P(C) = C \cdot d(t_1) + C \cdot d(t_2) + \cdots + (C + N) \cdot d(t_n)$$

Coupon of a new issue

$$C=\frac{P(C)-\frac{N}{(1+i_n)^{t_n}}}{\frac{1}{(1+i_1)^{t_1}}+\frac{1}{(1+i_2)^{t_2}}+\cdots+\frac{1}{(1+i_n)^{t_n}}}=\frac{P(C)-N\cdot d(t_n)}{d(t_1)+d(t_2)+\cdots+d(t_n)}$$

gdzie:
P(C) - cena obligacji kuponowej
t_1, t_2...t_n - czas trwania j-tego okresu (w latach), gdzie j=1,2,...n
d(tj) – czynniki dyskontowe dla czasu tj, gdzie j=1,2,...n
N – nominał
C – wartość kuponu,

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Example 5.2

The government wants to set an annual coupon rate for a 5-year bond (nominal is 100). Find the coupon if the following spot rates are available on the market: 1-year equal to 3.5%, 2-year 3.7%, 5-year 3.55%, 10-year 3.45%.

Find the coupon for 10 and 3 year issues.

The equation of the trend function is:

$$d(t) = 0{,}0007 \cdot t^2 - 0{,}0355t + 1$$

$$C = \frac{P(C) - N \cdot d(t_n)}{d(t_1) + d(t_2) + \cdots + d(t_n)} = \frac{100 - 100 \cdot 0{,}84}{4{,}51} = 3{,}55$$

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Part 6

MEASURES OF BOND'S RISK

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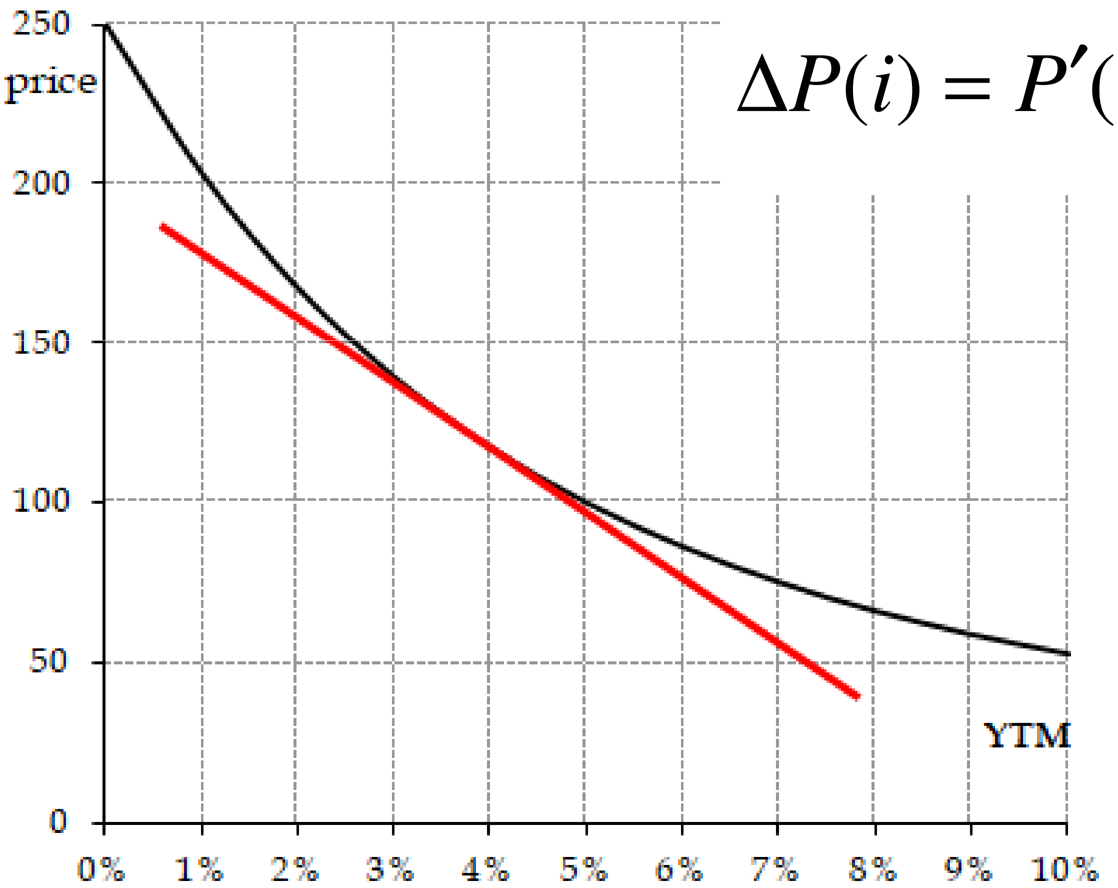
Price – yield relationship



$$\Delta P(i) = P'(i)\Delta i + \frac{1}{2}P''(i)\Delta^2 i + \ldots$$

Duration is a measure of the approximate percentage change in value for a 100 basic points change in rates

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Taylor's formula

$$f(x) = f(x_0) + \frac{\partial f(x_0)}{\partial x}(x - x_0) + \frac{1}{2}\frac{\partial^2 f(x_0)}{\partial x^2}(x - x_0)^2 + \ldots$$

$$P(i) - P(i_0) = \frac{\partial P(i_0)}{\partial i}(i - i_0) + \frac{1}{2}\frac{\partial^2 P(i_0)}{\partial i^2}(i - i_0)^2 + \ldots$$

$$\Delta P(i) = P'(i)\Delta i + \frac{1}{2} P''(i)\Delta^2 i + \ldots$$

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First derivative

$$P'(i)=\left(\sum_{t=1}^{n}\frac{C}{(1+i)^t}+\frac{N}{(1+i)^n}\right)'=$$

$$=C\left((1+i)^{-1}+(1+i)^{-2}+\ldots+(1+i)^{-n}\right)'+\left(N(1+i)^{-n}\right)'=$$

$$=C\left(-(1+i)^{-2}-2(1+i)^{-3}-\ldots-n(1+i)^{-n-1}\right)-nN(1+i)^{-n-1}=$$

$$=-\left(\frac{C}{(1+i)^2}+\frac{2C}{(1+i)^3}+\ldots+\frac{nC}{(1+i)^{n+1}}\right)-\frac{nN}{(1+i)^{n+1}}=$$

$$=-\frac{1}{(1+i)}\left(\sum_{t=1}^{n}\frac{tC}{(1+i)^t}+\frac{nN}{(1+i)^n}\right)\cdot\frac{P(i)}{P(i)}=-\frac{D}{(1+i)}P(i)$$

Definition of duration - I

$$P'(i) = -\frac{D}{(1+i)} P(i)$$

$$\Delta P(i) = P'(i)\Delta i$$

$$\Delta P(i) = -\frac{D}{(1+i)} P(i)\Delta i$$

Duration (D) specifies the approximate percentage change in price caused by the change in market rates

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Definition of duration - I

$$\frac{\Delta P}{P} = -\frac{D}{(1+i)}\Delta i \qquad D = \frac{1}{P}\left(\sum_{t=1}^{n}\frac{tC}{(1+i)^t} + \frac{nN}{(1+i)^n}\right)$$

D_M

$$\frac{\Delta P}{P} = -D_M \Delta i$$

Modified duration (D_M) allows to determine the approximate percentage change in price due to **a small change** in interest rates

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Definition of duration – II

$$D = \frac{1}{P}\left(\sum_{t=1}^{n} \frac{tC}{(1+i)^t} + \frac{nN}{(1+i)^n}\right) =$$

$$= \frac{1 \cdot C}{P(1+i)^1} + \frac{2 \cdot C}{P(1+i)^2} + \ldots + \frac{n \cdot (C+N)}{P(1+i)^n} =$$

$$= w_1 \cdot 1 + w_1 \cdot 2 + \ldots + w_n \cdot n = \sum_{t=1}^{n} t \cdot w_t$$

where $$w_t = \frac{1}{P}\frac{CF_t}{(1+i)^t}$$

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Definition of duration - II

$$D = \sum_{t=1}^{n} t \cdot \frac{CF_t}{P(1+i)^t}$$

Duration is the weighted average maturity of bond flows, where the weights are discounted flows to price.

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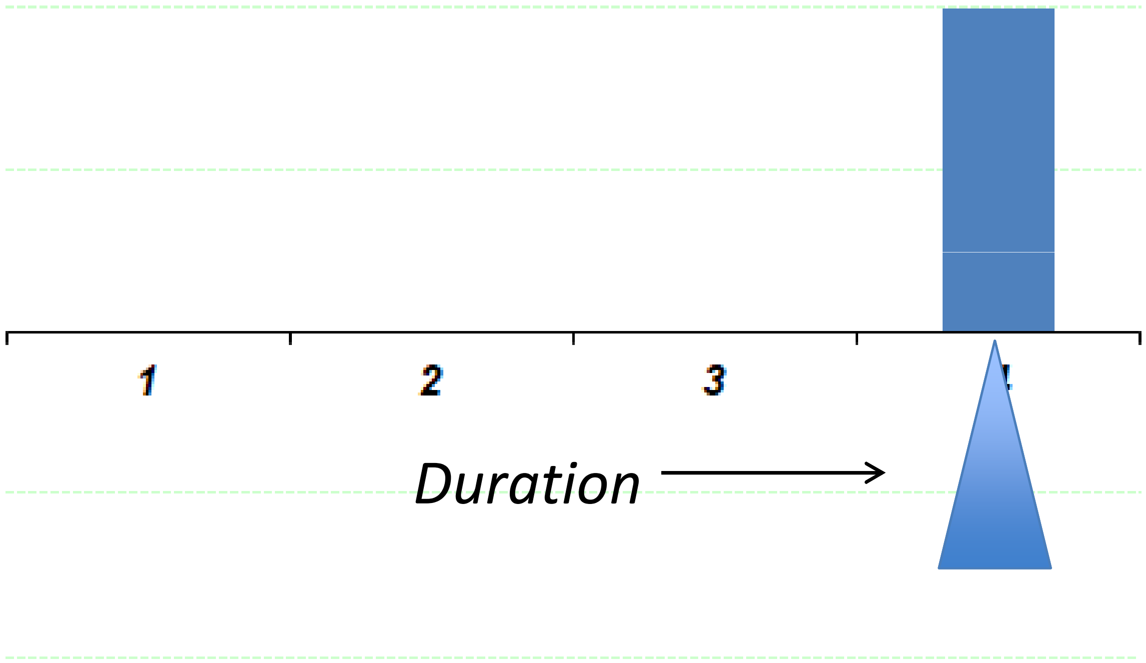
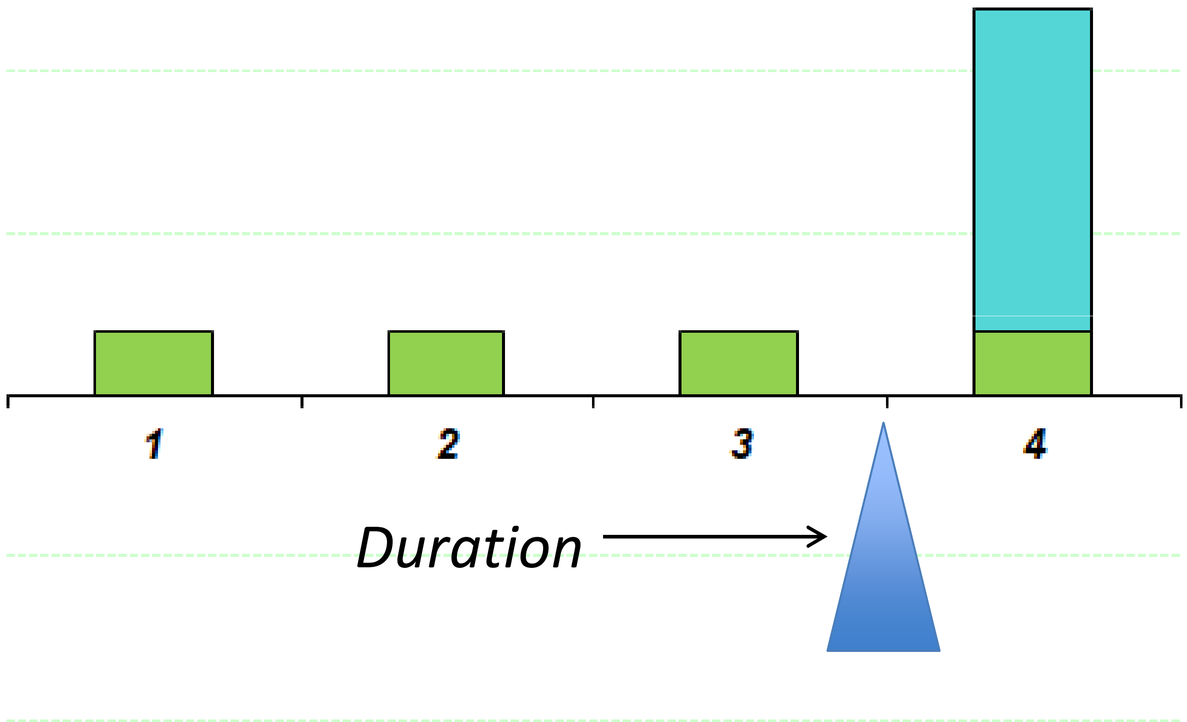
Definition of duration – II



For zero-coupon bond, a duration is equal to the maturity

Definition of duration – II



For coupon bond, a duration is lower then the maturity

Example

Find a duration, if the bond with yearly coupon 8%, and maturity of four years has YTM=10%, nominal 1.000

$$P = \sum_{t=1}^{n} \frac{C}{(1+i)^t} + \frac{N}{(1+i)^n}$$

$$P = 936{,}6027$$

$$D = \frac{1}{P}\left(\sum_{t=1}^{n} \frac{tC}{(1+i)^t} + \frac{nN}{(1+i)^n} \right)$$

$$D = 3{,}56$$

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Duration - features

1. Increasing maturity – increases duration
2. Increasing number of cash flows per year – reduces duration
3. Increasing yield to maturity – reduces duration
4. It is a good approximation of percentage price change for small changes in yield

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Duration - applications

1. Trading – estimating price changes for given yield changes
2. Hedging - estimating **relative** price changes for given yield changes
3. Risk management – matches volatility of assets and liabilities
4. Portfolios – providing **single measure** of risk for a portfolio
5. Investing – locking in guaranteed returns

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Second derivative

$$\Delta P(i) = P'(i)\Delta i + \frac{1}{2}P''(i)\Delta^2 i + \dots$$

$$P'' = \left(-\frac{C}{(1+i)^2} - \frac{2C}{(1+i)^3} - \dots - \frac{n(C+N)}{(1+i)^{n+1}} \right)' =$$

$$= \left(-C(1+i)^{-2} - 2C(1+i)^{-3} - \dots - n(C+N)(1+i)^{-n-1} \right)' =$$

$$= 2C(1+i)^{-3} + 2 \cdot 3 \cdot C(1+i)^{-4} + \dots + n(n+1)(C+N)(1+i)^{-n-2} =$$

$$= \sum_{t=1}^{n} \frac{t(t+1)C}{(1+i)^{t+2}} + \frac{n(n+1)N}{(1+i)^{n+2}}$$

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Convexity - definition

$$P'' = \sum_{t=1}^{n} t(t+1)\frac{CF_t}{(1+i)^{t+2}} \cdot \frac{P(i)}{P(i)} = CX \cdot P(i)$$

$$CX = \frac{1}{P}\sum_{t=1}^{n} t(t+1) \cdot \frac{CF_t}{(1+i)^{t+2}}$$

Convexity (CX) allows to determine the approximate percentage change in price due to **a bigger change** in interest rates

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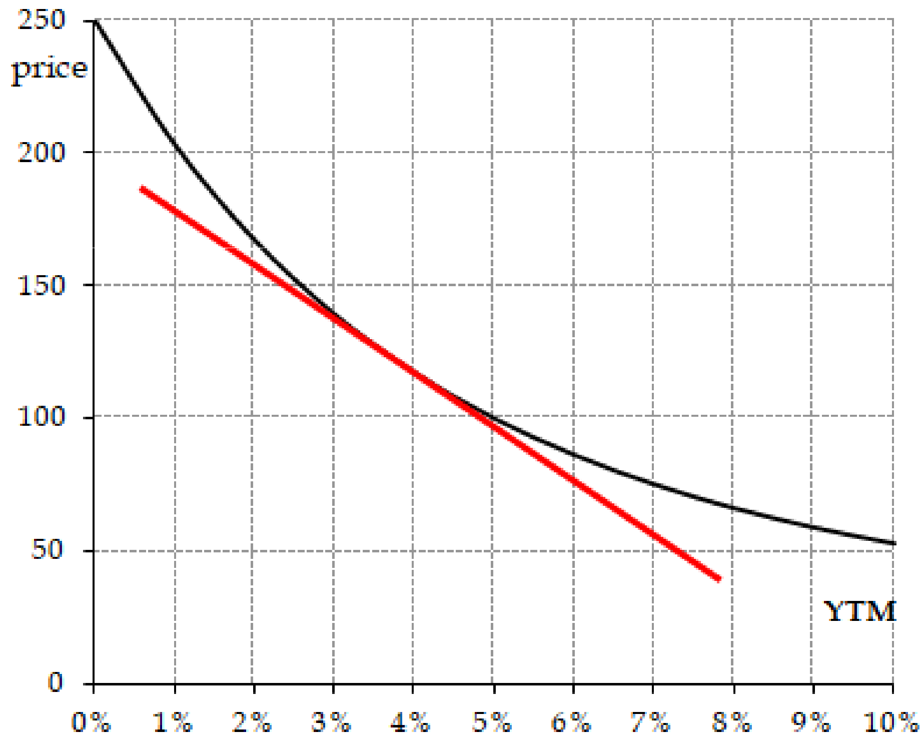
Convexity – applications

$$\Delta P(i) = P'(i)\Delta i + \frac{1}{2}P''(i)\Delta^2 i + \ldots$$

$$P''(i) = CX \cdot P(i)$$

$$\Delta P(i) = -D_M P(i)\Delta i + \tfrac{1}{2} CX \cdot P(i)$$

$$\boxed{\frac{\Delta P(i)}{P(i)} = -D_M \Delta i + \frac{1}{2} CX \cdot \Delta^2 i}$$



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Convexity – interpretation

$$\frac{\Delta P}{P} \cong -D\Delta i + \frac{1}{2} CX\left(\Delta i\right)^2$$

If two bonds have the same duration, the one with more convexity is normally preffered

Two portfolios with similar durations may perform quite differently if the yield curve shifts in a nonparallel fashion.

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Example

A 30-year bond with 8% coupon is sold with 8% yield (price is 1,000); D_M = 11.26; CX = 212.4.

If the rate increases by 2%, then the price will drop to 811.46. What decrease could be predicted by duration, and what by a compilation of D_M + CX

$$\text{price's decrease} = \frac{\Delta P}{P} = 18{,}85\%$$

$$\frac{\Delta P}{P} = -D \cdot \Delta i = -11{,}26 \cdot 0{,}02 = -0{,}2252$$

$$\frac{\Delta P}{P} \cong -D\Delta i + \frac{1}{2} CX(\Delta i)^2 = -0{,}2252 + \frac{1}{2} 212{,}4(0{,}02)^2 = -0{,}1827$$

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Example

A 30-year bond with 8% coupon is sold with 8% yield (price is 1,000); D_M = 11.26; CX = 212.4.

If the rate increases by only 0.1%, then the price will drop to 988.85. What decrease could be predicted by duration, and what by a compilation of D_M + CX

$$\text{price's decrease} = \frac{\Delta P}{P} = 1{,}115\%$$

$$\frac{\Delta P}{P} = -D \cdot \Delta i = -11{,}26 \cdot 0{,}001 = -0{,}01126$$

$$\frac{\Delta P}{P} \cong -D\Delta i + \frac{1}{2} CX (\Delta i)^2 = -0{,}01126 + \frac{1}{2} 212{,}4 (0{,}001)^2 = -0{,}01115$$

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Convexity – conclusion

1. Increasing maturity – increases convexity / duration
2. Increasing number of cash flows per year – reduces convexity / duration
3. Increasing yield to maturity – reduces convexity / duration

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